TUB-Relativity Model of Colour Vision for Light and Surface Colours

Prof. Dr. Klaus Richter, Technische Universität Berlin (TUB), 22-07-21
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Summary
The TUB-relativity model of colour vision is based on research on colour vision and many visual and psychophysical experiments with light and surface colours. The TUB model can be used to describe colour attributes, such as chroma, lightness and hue, chromatic adaptation, colour thresholds, colour differences and many visual phenomena, for example the colour consistency of the CIE illuminants D65, D50, A and E. The TUB model is suitable for applications in image technology, for example for colour output on displays or in printing as well as for photographic or scanner colour input.

CIE colourimetry is mainly based on experiments with surface colours, for example on colour attributes and colour differences. The six *Ostwald optimal colours RYGCBMs* are the most chromatic colours. The three *complementary Ostwald* optimal colour pairs *R-C, Y-B,* and *R-M* mix to white.

Light colours, for example of *sRGB* and *OLED* displays, are more narrow band compared to the *Ostwald-optimal colours*. Even with display colours, the colour pairs *R-C, Y-B*, and *R-M* mix to white. Physically, they can be described as narrow band optimal colours. Ostwald-optimal colours and the *sRGB* display colours consist of six colours *RYGCBM.* For both systems, all chromatic values C_{A2B2} are approximately equal in a hue hexagon. For the complementary optimal colours, the chromatic values of *R and C, Y and B*, as well as *R and M* are antagonistic and their two chromatic values C_{A2B2} have the same amount.

According to *Holtsmark* and *Valberg* (1969), the colour discrimination is approximately the same for all complementary optimal colours. This experimental result is a special basis for expanding the CIE colourimetry for surface colours to a TUB colourimetry for *light and surface colours* in this work.

For many applications, the metric of the TUB-relativity model of colour vision for *light and surface colours* also improves the previous CIE metric for *surface colours* as a subset.

An introduction shows a colour loop:

ISO-Colour File – ColourFileOutput – ColourTestChartInput – ISO-Colour File

The *rgb** colour data in a digital ISO-colour file should be approximately the same for each software and hardware combination at the end.

This goal can be achieved with a suitable metric for *light and surface colours*, for example with the TUB metric for *light colours* of displays. ISO 9241-306:2018 uses this metric for the display colour output and defines for applications eight different display contrasts.

Notes:
Many applications in ISO, IEC, CEN, CIE and DIN standards and research results are available on a server of the Technische Universität Berlin (TUB) in English, German and partly in four other languages, see http://color.li.tu-berlin.de/index.html

1. Introduction: Colour loop with *light and surface colours*

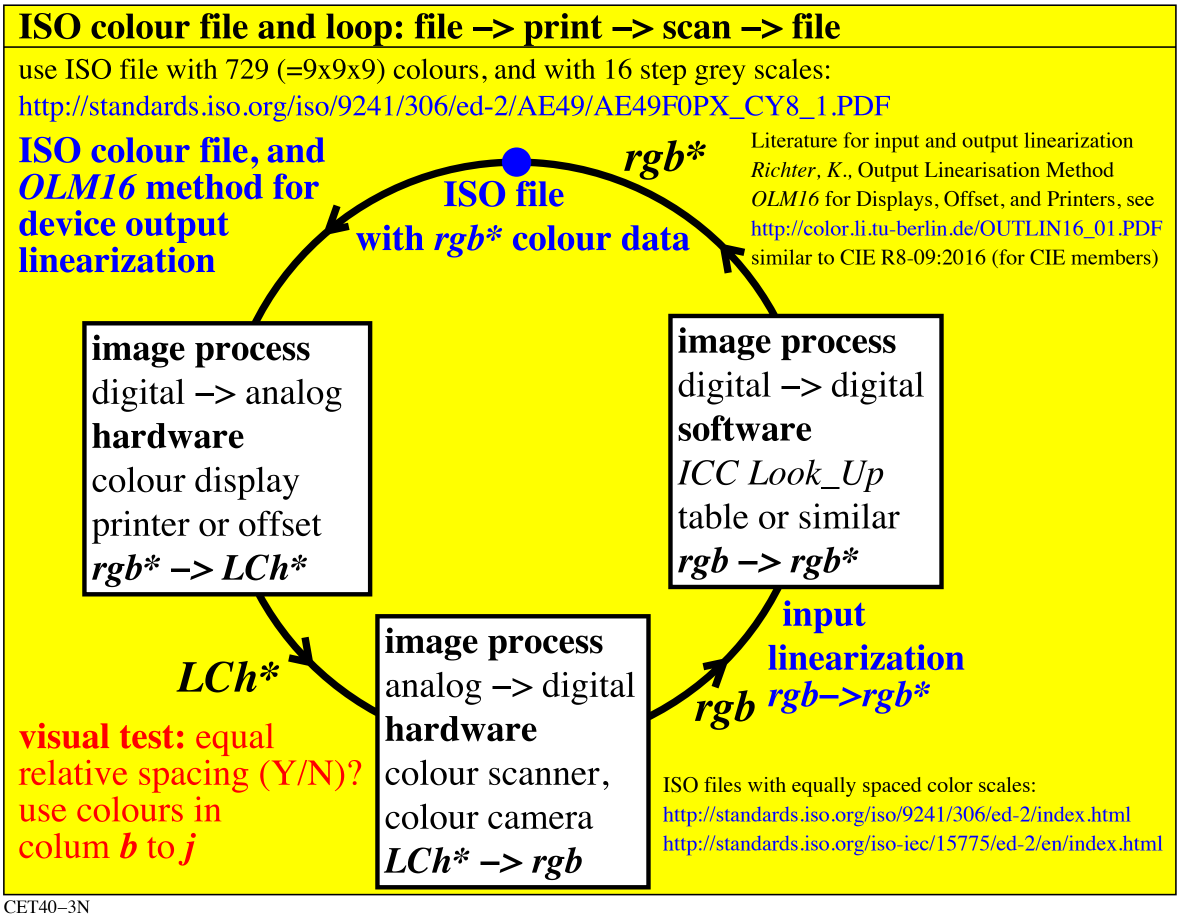


Figure 1: Colour loop: ISO file -> File output -> Test chart input -> ISO file.
For the download of this figure, see http://color.li.tu-berlin.de/CET4/CET40-3N.PDF

In information technology, digital and analog ISO-test charts are used to realize the colour loop of Figure 1.

The *rgb** colour data in an ISO file should be approximately equal for each software and hardware combination at the end of the loop compared to the beginning of the loop. This goal can only be achieved with an appropriate metric for *light and surface colours*.

A special TUB colour model has been developed for *the light colours* of displays. It can also describe the *Ostwald*-optimal colours. These optimal colours are the surface colours with the largest chromatic value and chroma. Therefore, TUB colourimetry is suitable for many applications. Special examples are presented in this work.

Figure 1 contains links to download the digital ISO-test charts of the ergonomic standard ISO CEN DIN 9241-306/ed-2:2019 for the display output. The standard ISO/IEC 15775/ed-2:2022 contains similar analog (printed) ISO-test charts for colour copiers. Both standards and the DIN 33866-1 to 5 series also contain output questions for visual inspection in English, German and four other languages for free download for all colour devices of the colour loop.

The colour loop is of particular importance in the application. For example, for the archiving of works of colour art and colour documents and their reproduction in museums on colour displays.

2. Reproduction of works of colour art and colour documents in museums on displays

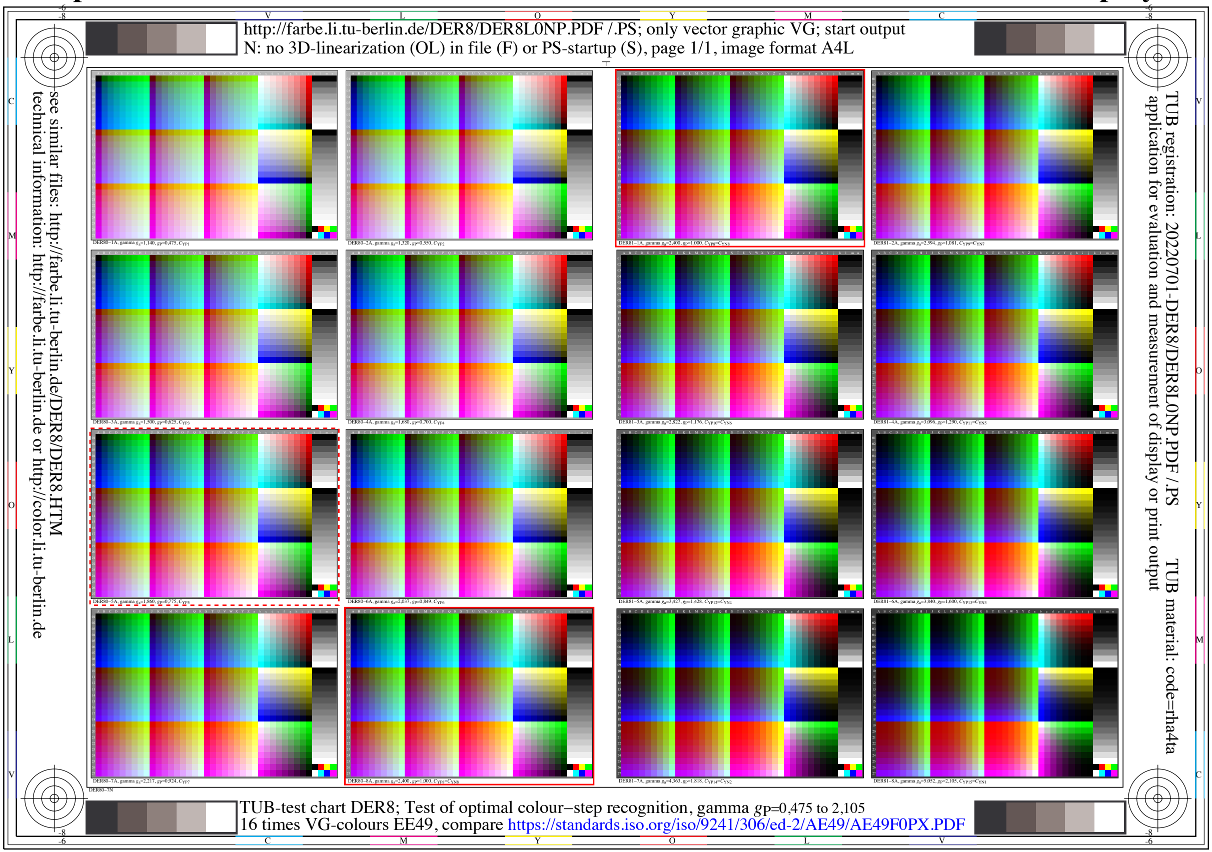


Figure 2: Equally stepped colour series in *rgb and their visual reproduction with 16 gamma values according to ISO 9241-306:2018.**
For the download of this figure, see http://color.li.tu-berlin.de/DER8/DER8L0NP.PDF

Figure 2 shows 16 times the 1080 colour samples of ISO CEN DIN 9241-306:2019. The 16 images are distinguished by the *relative* gamma values 0.445 <= g_P <= 2.11. Between the *relative* and *absolute* gamma value , the following equation applies: $g_a = 2.4\ g_P$.
For g_P=1 , *the* result is g_a=2.4. The absolute gamma value creates a linear relationship between the chromatic values "*rgb"* in a digital file and the CIE lightness L^*_{CIELAB}. The following applies in the *sRGB* colour space according to IEC 61966-2-1:

$$rgb^* = L^*_{CIELAB}/100 \text{ with } 0 <= rgb^* <= 1 \text{ and } 0 <= L^*_{CIELAB} <=100. \quad [1]$$

The coordinates rgb^* are proportional to the sensation lightness L^* and therefore receive an asterisk (*). The coordinates proportional to the physical relative luminance are specified with *rgb*. The following applies:

$$rgb^* = rgb^{(1/2.4)}. \quad [2]$$

Equations [1] and [2] require a linear relationship between digital file data rgb^* in the range 0 to 1 and the *relative* lightness $L^*_{CIELAB}/100$ in the range 0 to 1.

In Figure 2, the gamma values vary in the range 0,445 <= g_P <= 2,11 or 1,07 <= g_a <= 5,06. At the VDU workstation, the gamma value can change from g_a=2,4 to almost g_a=1,0 due to the increasing reflection of the ambient light on the display.

Figure 2 simulates the change of colours and their gradation. At the top left of Figure 2, for example, the light grey steps are *indistinguishable*. At the bottom right of Figure 2, the dark colour steps are *indistinguishable*. ISO 9241-306 describes a gamma slider that makes all 16 grey steps visible again, see also the following picture 9. For this purpose, a gamma in the range 1,0 < = ga < = 2,4 is used in the office. The value required for the ergonomic output depends on the hardware and software of the display and the reflection.

3. White foot and contrast as the basis for the TUB-relativity model of colour vision
It is believed that the visual system of man constantly adapts to a "medium" achromatic stimulus with a "medium" luminance of the visual scene a. Based on the TUB colour vision modell, all light and surface colour sensations are calculated relative to these "average" measurable data of colour and light.

Colour stimuli are described a *linear mean* and colour sensations approximated by a *logarithmic mean*. For small contrasts and colour differences, the linear and logarithmic mean is approximately proportional. Therefore, for small contrasts and colour distances, *linear* colourimetric formulas for describing visual results are also suitable.

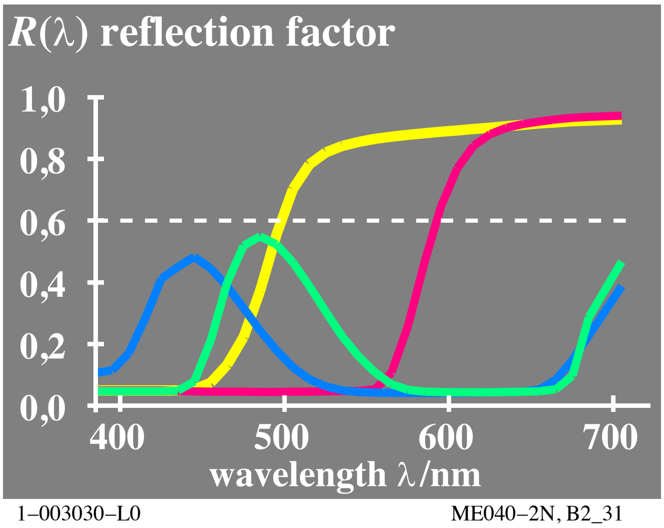


Figure 3: Reflection of tsurface colours for the CIE test colours 9 to 12 according to CIE 13.3.
For the download of this figure, see http://color.li.tu-berlin.de/ME04/ME040-2N.PDF

The surface colours of figure 3 and all other *matte* surface colours have a basic reflection close to 3,6%. This basic reflection is called white foot. The white foot with the same reflection at all wavelengths contains *no* colour information and is therefore usually subtracted in the TUB-relativity model of colour vision to calculate the chromatic colour attributes.

The TUB model therefore has a fundamental difference from the description of the colour attributes compared to the CIELAB colour space. Normally, the TUB model takes into account the scattered light produced by the lens of the eye and the eye media with a value of at least 1%. This value increases with increasing age, for example to the value of 3,6%.

Therefore, a reflection below 1% is not realistic in many applications. Both the reflections of the surface colours or light colours as well as those of the eye media must be taken into account. Therefore, in many applications, a reflection of 3,6% is appropriate.

With the reflection close to 90%f of white according to the ergonomic standard ISO 9241-306 the contrast between white and black is C=90/3,6=25:1. The contrast does *not* change for surface colours, if for example the office illuminance is increased from 500 lux to 5000 lux.

However, the contrast may reduce by a factor 10 with a 16-step grey scale between white and black on an office displays. For example the contrast reduces from C=25:1 to C=2:1 for an increase of the office illuminance from 500 lux to 5000 lux. This increases the display reflection by a factor 10 and reduces the contrast by a factor 10.

3. Colour stimuli and colour sensation for chromatic colours: mixing and antagonism

The most chromatic surface colours are the *Ostwald* colours. In many applications, three colour pairs *red-cyan* (R-C), *yellow-blue* (Y-B) and *green-magenta* (G-M) are used.

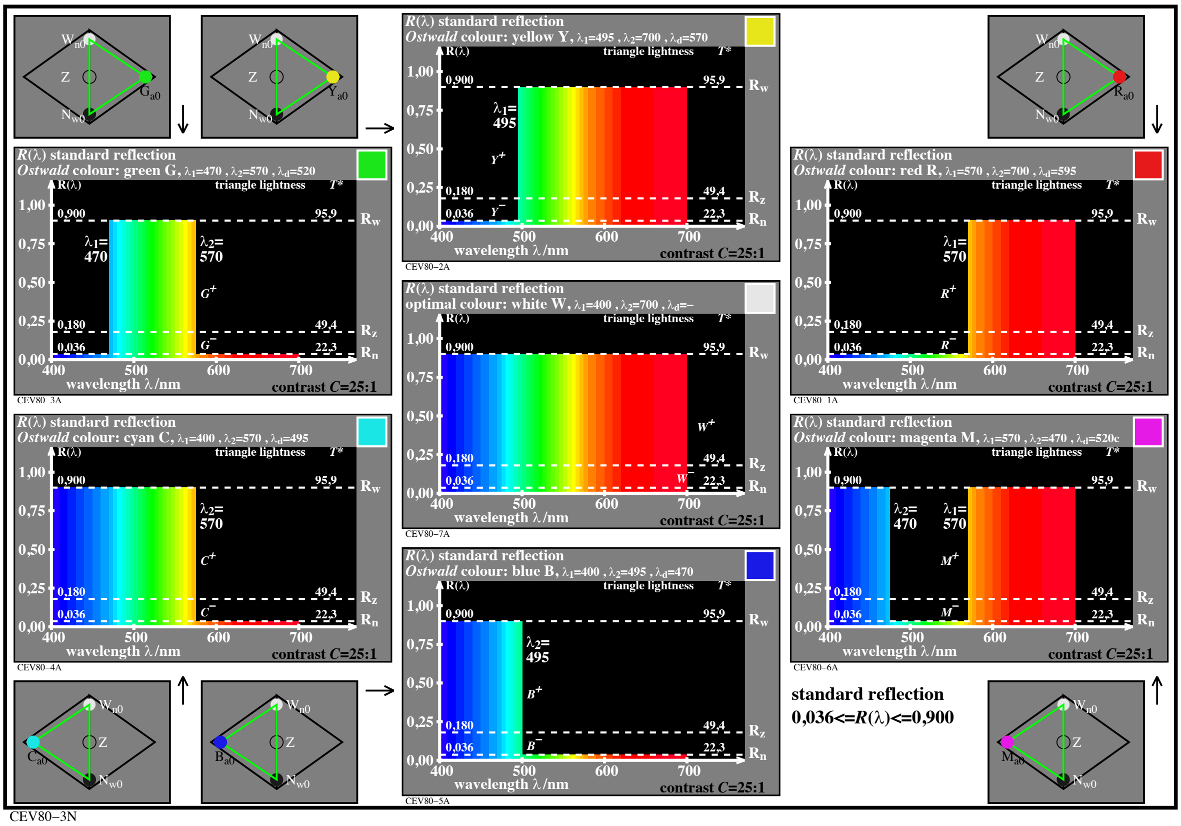


Figure 4: Reflections as a function of the wavelengths of six *Ostwald* colours and the position of their mixed colours in different hue planes.

The mixed colours are shown in the upper right corner of the seven partial figures.
For the download of this figure, see http://color.li.tu-berlin.de/CEV8/CEV80-3N.PDF

Figure 4 shows the *Ostwald* colours and its reflections in a 6-step colour wheel *RYGCBM*. The values of the standard reflections R_W of white, R_Z of medium grey and R_N of black (N=noir) are by a factor of 100 smaller than the standard chromatic values Y_W, Y_Z and Y_N . In the TUB model the white foot has the reflection R_N=0.036. The middle grey has a reflection of R_Z = 0.180. The white has a reflection of R_W = 0.900. The three values are shown in Fig. 4 by dashed lines. The relative reflections $R_r = R/R_Z$ are in the range

$$0.2 <= R_r <=5 \quad [3] \text{ or.}$$

$$-0.69<=\log R_r<=0.69 \quad [4].$$

The *relative* lightness function compared to the medium grey Z

$$L^*_{TUBr} = k \log R_r. \text{ with } k = 40/\log(5) = 57.22 \qquad [5]$$

meets a standard goal of the TUB colour vision model .
Similar to the OSA colour system, the relative lightness is zero for medium grey Z and negative for darker colours and positive for lighter colours. The lightness L^*_{TUBr} is an S-shaped function and saturates below black *N* and above white *W*.
In the TUB model, the lightness values are therefore for white (W), grey (Z), and black (N):

$$L^*_{TUBrW}=40,\ L^*_{TUBr,Z}=0\ ,\ L^*_{TUBr,N}=-40 \quad [6]$$

If the value 50 for grey Z is subtracted from L^*_{CIELAB}, the *relative* CIE lightness is:

$$L^*_{CIEr} = L^*_{CIELAB} - 50 \quad [7]. \text{ The three values are:}$$

$$L^*_{CIEr,W}=45,\ L^*_{CIEr,Z}=0,\ L^*_{CIEr,N}=-28. \quad [8].$$

The symmetric relative lightness L^*_{TUBr} [5] *is a logarithmic function.*

4. Colour sensation for chromatic colours: mixture, signals and antagonism

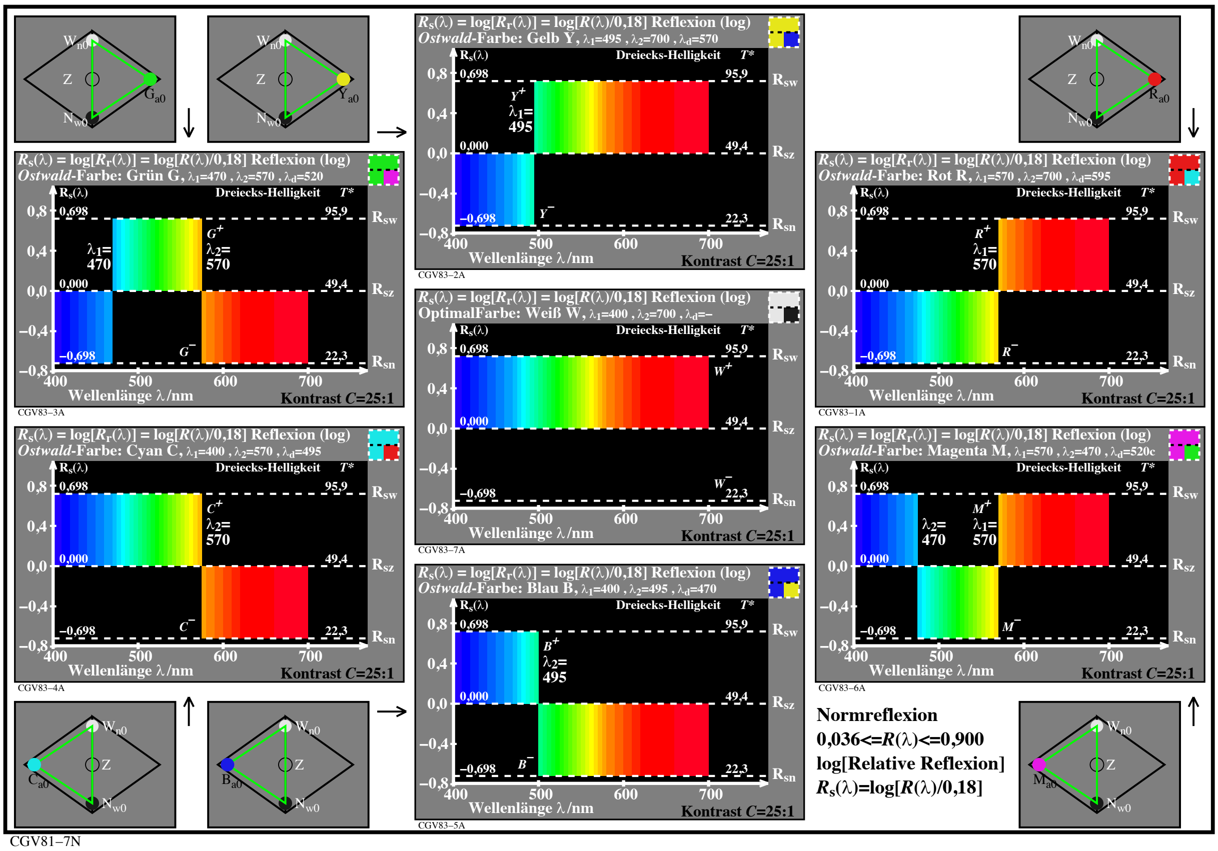


Figure 5: Antagonistic signals R^+ and R^- of six *Ostwald* colours and their positions in different hue planes.

For the download of this figure, see http://color.li.tu-berlin.de/CEV8/CEV81-7N.PDF

For the calculation of the antagonistic signals R^+ and R^- and for the five hues *YGCBM* , the logarithmic *relative* reflections $R_r=R_W/R_Z$ und $R_r=R_N/R_Z$ are used. No physical reflection smaller than R_N of black is used for the chromatic colour signals. We see a main feature: any reflection smaller than R_N does not contain colour information, see Figure 3. Therefore, the reflection range between 0 and R_N is not used to calculate the *colour values*. This is a fundamental difference between a relative TUB and CIE colourimetry.

For example, the relative CIE colourimetry of CIELAB includes the reflection part R_{rN}=0.0 36 down tor relative reflection R_r=0.0. For matte samples, the limit value is R_{rN}=0.036. In application, retroreflective white samples may have a higher R_r value. Similar glossy black samples may have a lower value R_r.

In the TUB model, the maximum reflection data of the colour pairs *R-C, Y-B* and *G-M* are the same and equal to the white reflection R_w=0.900 with the relative reflection R_{rw}=5. The chromatic values C_{A2B2} are the same and the tristimulues values *Y* differ. They have the highest value near yellow and the lowest value near blue, see the *Y* values in Figure 5.

For example, the CIELAB chroma C^*_{ab} and the *RG* and *YB* components a^* and b^* of all *Ostwald* colour pairs differ. In the TUBr model, however, the chromatic values of the components *RG* and *YB* are, as expected, antisymmetric because they mix to a grey.

5. Equal chromatic values C_{A2B2} and equal hue-angle discrimination Δh_{A2B2}

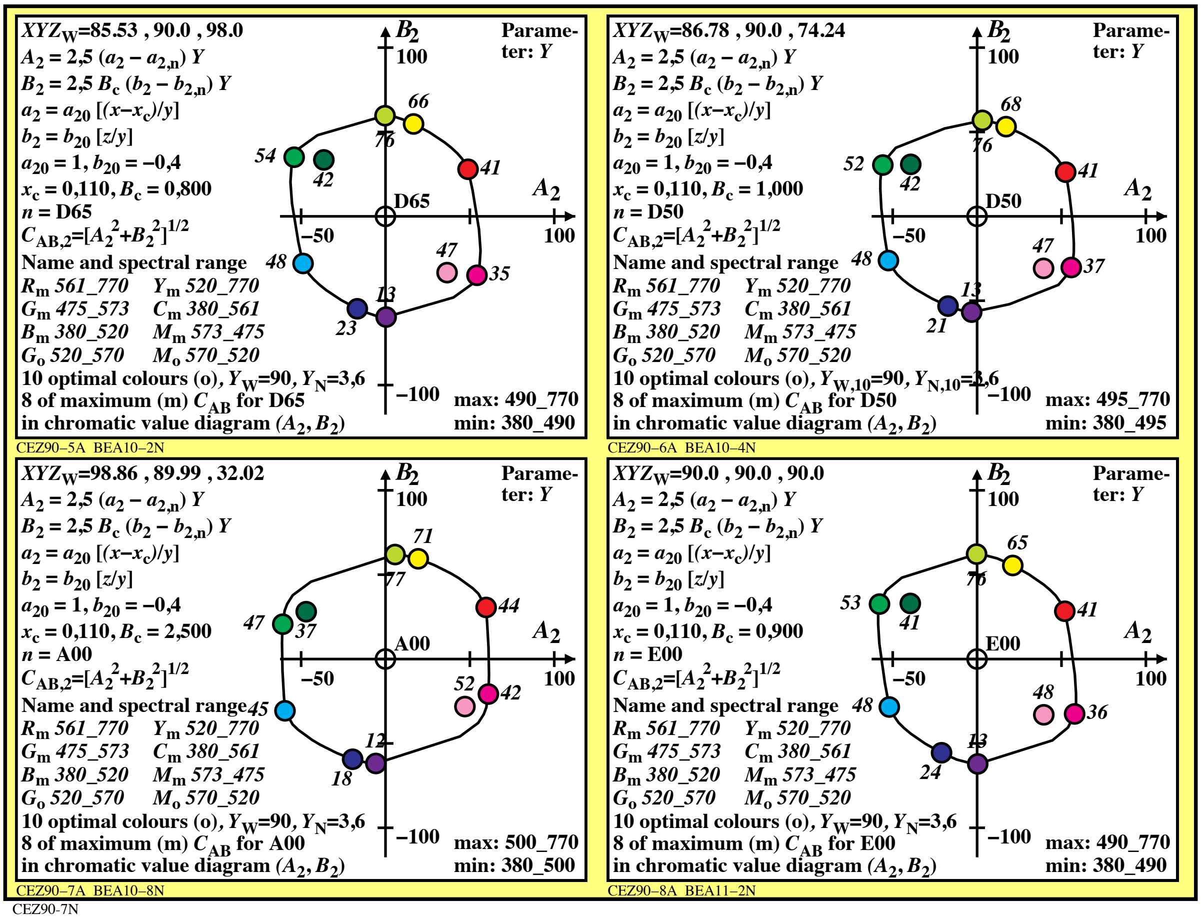


Figure 6: Chromatic values of 6 *Ostwald* colours for CIE illuminants D65, D50, A, and E
For the download of this figure, see http://color.li.tu-berlin.de/CEZ9/CEZ90-7N.PDF

For the *Ostwald* colours the CIE chromatic values were calculated for four different CIE illuminants. The chromatic values C_{A2B2} are determined from the CIE values *x, y, Y* and an adaptation parameter B_c. It applies to B_c: 0.8 for D65, 1.0 for D50, 2.5 for A and 0.9 for CIE illuminant E. The chromatic-value components A_2 and B_2 have the same amount for all colour pairs. Therefore, the radial chromatic value C_{A2B2} is the same for all colour pairs. In addition, the radial chromatic value is also approximately the same for *all Ostwald* colours. This is expected according to the *law of colour consistency* for a change of the illuminant. All values of the TUB model are *antagonistic and antisymmetric*. Therefore, one can assume that *equal* hue angle changes Δh_{A2B2} produce *the same* visual differences.

For the *Ostwald* colour pairs the CIELAB chroma *a** and *b** differ by a factor of 2 (not shown here). Therefore the CIELAB chroma values are *not* antisymmetric and approximately equal. This are properties for the *linear* chromatic values A_2 and B_2 of the TUB model. The *linear* chromatic values of the TUB model probably also give a good description of the *visual* chroma.

Even if *nonlinear* formulae for the transfer from the chromatic values to the chroma values are used, then equal chroma values are expected. For example the TUB model with *linear* chromatic values can be extended by a *logarithmic transfer* which is *linear* for small chroma.

Note: Some wavelength boundaries of the *Ostwald* colours are slightly different compared to Figures 3 and 4. The wavelength limits change by 5 nm with the CIE illuminant, see in Fig. 6 the ranges after *max* right down for the CIE illuminants D65, D50, A and E.

6. Colour hue discrimination as function of the hus angle Δh_{A2B2} and the wavelength $\Delta\lambda$

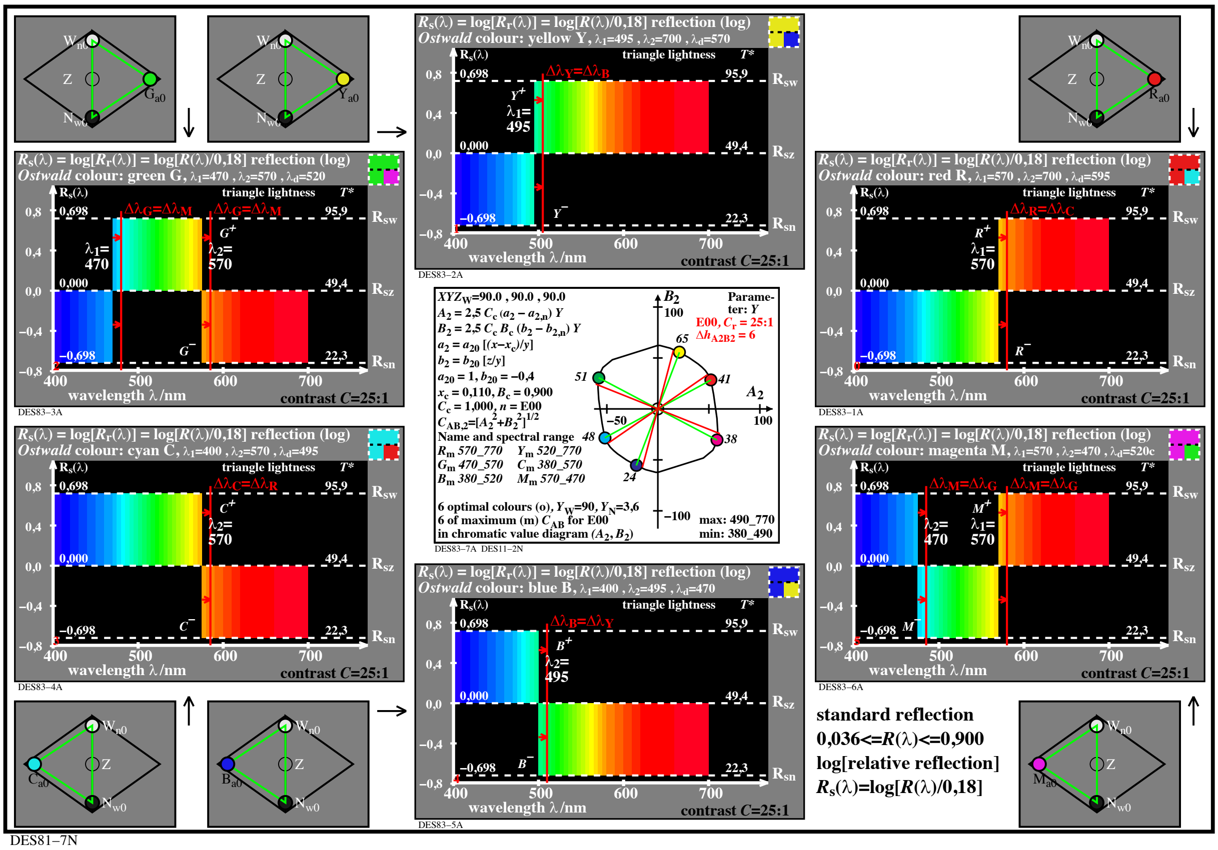


Figure 7: Spectral and chromatic values of six *Ostwald* colours for the CIE illuminant E with hue discrimination as a function of the hue angle Δh_{A2B2} for the contrast 25:1.
For the download of this figure, see http://color.li.tu-berlin.de/DES8/DES81-7N.PDF

The spectral colour mixture in Fig. 7 produces the most chromatic surface colours. They are shown in the upper right corner for the six colours together with complementary colours. According to the psycho-physical experiments of *Holtsmark and Valberg* (1969), the threshold of wavelength shift Δλ of complementary optimal colours *R-C, Y-B,* and *G-M* is approximately the same, see the six *Ostwald* spectra. The shift Δλ is experimentally smaller for the pair *Y-B* than for the other two pairs (not shown here). In Fig. 7 it is assumed that the colour threshold (a just perceptible difference JND) is about 6 degrees for the 6 colours of the *Ostwald*-colour circle. Therefore, about 60=(360/6) steps of the very chromatic *Ostwald* colour circle can be distinguished.

Figure 7 shows in the middle part a hue-angle change Δh_{A2B2}=6 degrees due to red lines from the origin compared to the green lines. The six red lines correspond to six red lines in the six *Ostwald* spectra. The TUB model results in the *same* hue-angle shift Δh_{A2B2}=6 for *all* six *Ostwald* colours. This prediction is to be expected due to the approximately constant chromatic values for all shifts. The only slight change in lightness compared to the chromatic value will hardly affect the threshold.

Holtsmark and Valberg (1969) used adjacent optimal colours in a white environment and in a dark room. This white-black state can lead to a medium grey adaptation. For different adaptations, the value Δλ will only change by a constant factor. For purple *Ostwald* colours, the complementary wavelength λ_{1c} and λ_{2c} of *Holtsmark and Valberg* are given.

Holtsmark, T. and Valberg, A. (1969), Colour discrimination and hue, Nature, Volume 224, October 25, pp. 366-367.

7. Experiment for equal hue discrimination of complementary optimal colours
There is a visual phenomenon with complementary optimal colours produced by a positive and negative mask. A prism visually creates a similar distinction at corresponding locations within both spectra. This indicates a high symmetry in vision and is a basis for the TUB colour vision model. In addition, this leads to improved formulae for describing the colour threshold, see CIE 230:2019 entitled: *Validity of Formulas for Predicting Small Colour Differences.*

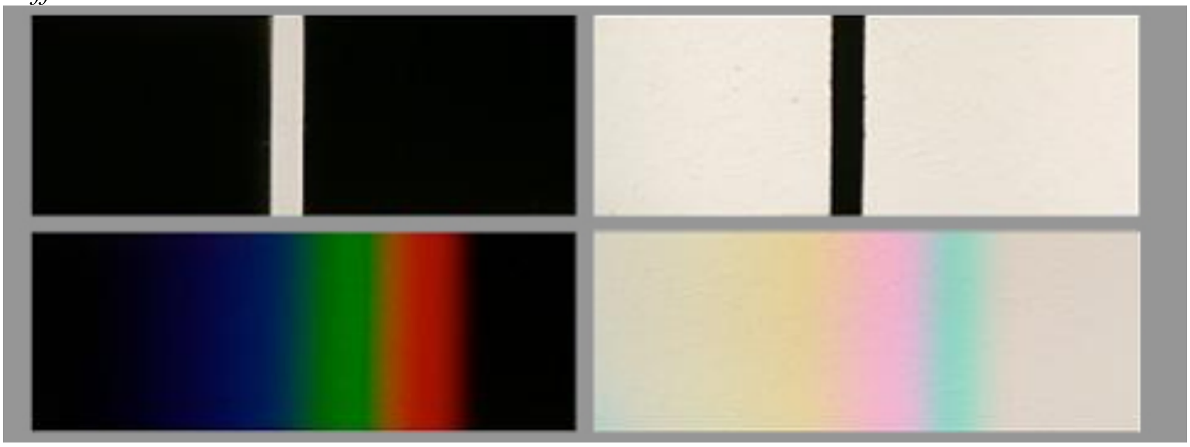


Figure 8: Complementary optimal colours produced by a positive and negative mask with a prism.

Figure 8 shows optimal colours produced by a mixture of all spectral colours between two wavelength limits. If the wavelength range for both the positive and negative mask is about 100 nm, we see in Figure 8 (left) three primary colours *O, L* and *V* (orange red, leaf green and violet blue according to ISO/IEC 15775:2022) and im Fig. 8 (right) three complementary colours *C, M* and *Y* (cyan blue, magenta red and yellow). The visual phenomenon arises, since the visual hue discrimination for the two colour series *OLV* and *CMY* for corresponding places is approximately the same. This applies approximately to each mask width for all positive and negative masks and therefore for *all* complementary optimal colours.

Holtsmark and Valberg (1969) measured the hue discrimination for optimal colours with a spectral colour integrator. In a white surround in a dark room with two identical masks, they have created two equal semi-circular central fields of about 2 degrees in diameter. The position of one mask had to be shifted until a hue threshold was visible. The same was repeated through an inverse mask and for six observers. The summary is in the statement: *The hue discrimination is approximate equal for a negative and positive mask.*

The colour change is mainly defined by a constant hue angle shift, for example Δh_{A2B2} =6, see the chromatic values in Fig. 7. In the experiments of *Holtsmark and Valberg*, the chromatic values C_{A2B2} change with the gap width. Therefore, the experimental results of *Holtsmark and Valberg* are more general: *All complementary optimal colours* and not only complementary *Ostwald* colours produce the *same* experimental colour discrimination. The TUB colour vision model forms a basis for describing these additional experimental results. For this purpose, experimental results for the change of lightness and chromatic values as function of contrast are used in the following.

Acknowledgements: I would like to thank Prof. Dr. Johannses Grebe-Ellis, the owner of the website http://www.experimentum-lucis.de, *for the permission to use in this work a figure* of *his website (which contains many more references*).

8. Change of lightness and lightness threshold for the contrasts *C*=90:1 and 2:1

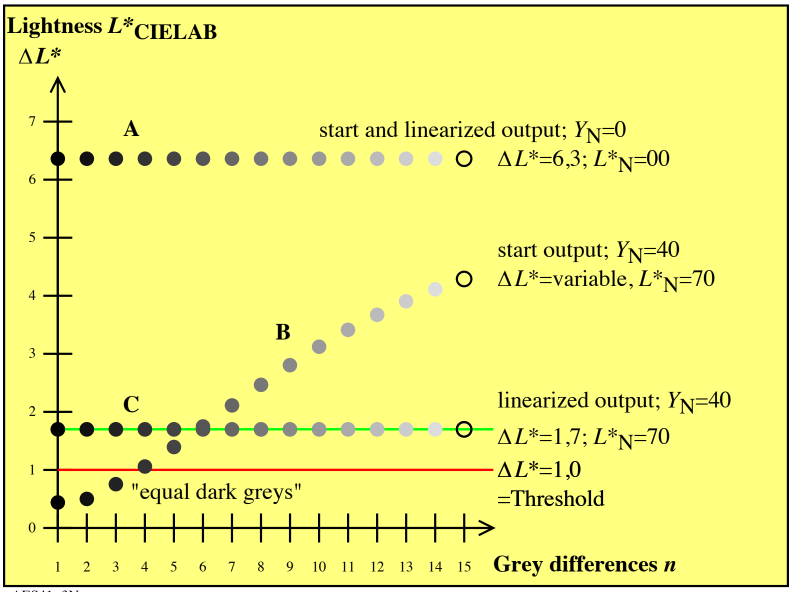


Fig. 9. Lightness differences ΔL^*_{CIELAB} of a 16-step grey scale on the *sRGB* display for high contrast *C*>25:1 and small contrast *C*=2:1.
For the download of this figure, see http://color.li.tu-berlin.de/AES4/AES41-3N.PDF

According to Figure 9, the lightness difference is ΔL^*_{CIELAB} = 6.3(=100/15) for a 16-step grey scale. This value results according to ISO 92421-306 for an *sRGB* display according to IEC 61966-2-1 in the dark room (without room light reflection and without eye media reflection). The contrast $C=Y_W/Y_N$ is in this ideal case because of Y_N=0 for black *N* (=noir) infinitely large. However, in the application in the office with the illuminant 500 lux and the room light reflection 3,6% on the display, the contrast *is* *C*=90:3,6=25:1.

In some cases, the contrast decreases to *C*=2:1, for example for $C=(L_P+L_D)/L_D$. In this case, the luminance L_P from a digital projector and the luminance L_D from daylight (D=Daylight) on the projection screen is the same.

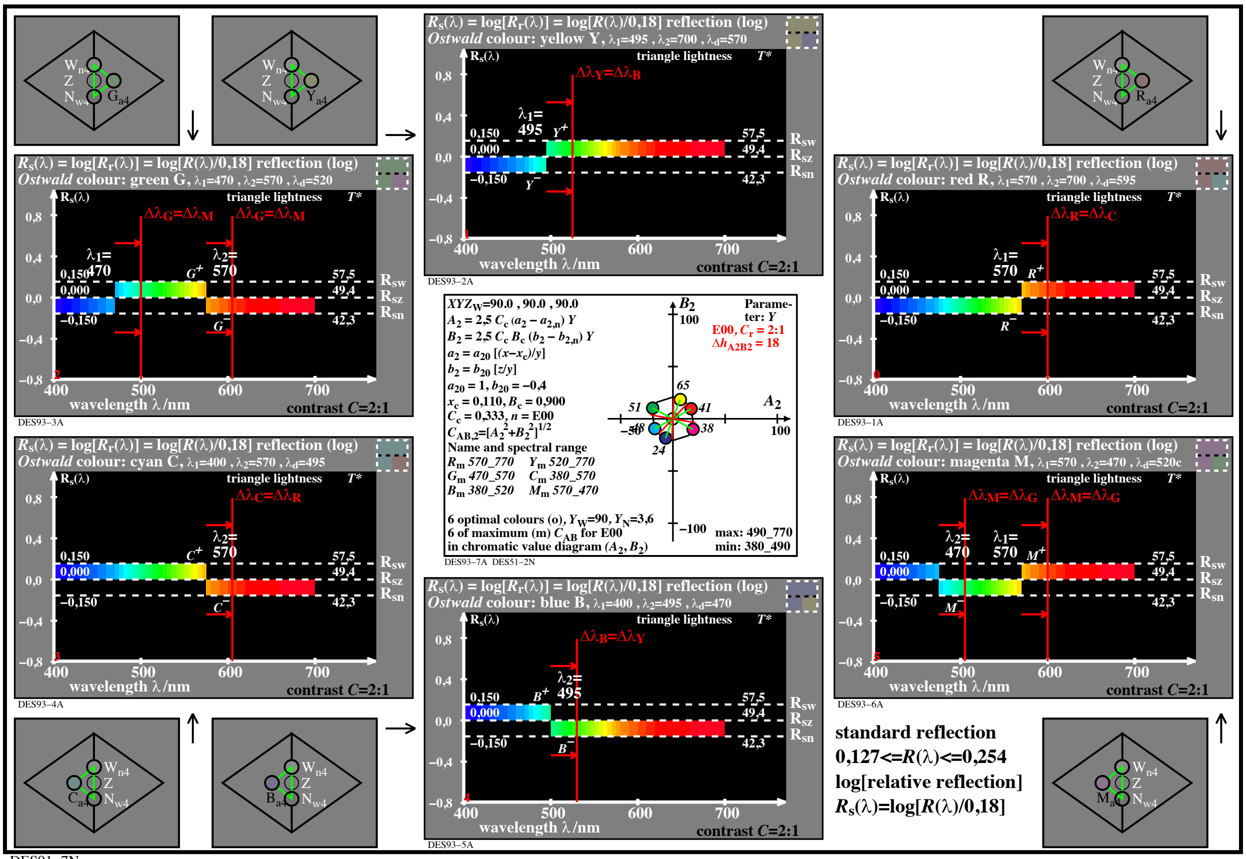
According to Figure 9 , in this case the differences in lightness of the grey scale samples decrease compared to ΔL^*=6.3 and are *no longer equal.* For four dark steps, the calculated lightness differences are smaller than the lightness threshold. Therefore, 4 out of 16 grey samples are visually indistinguishable.

ISO 9241-306 describes as an example a gamma slider on the *Mac* operating system, which makes all 16 steps with the constant smaller lightness difference ΔL^*=1.7 visible again. This fulfils the special condition of ergonomics for the visibility of all 15 differences. In the *Windows* operating system, there is currently no corresponding method. However, application programs fulfil the same task. On both systems, for example, *ICC* profiles with 8 defined gamma values can be used.

With a reflection of 1% of the eye media compared to white, the contrast can be approximately *C*=90:1 and the lightness difference may be ΔL^*=5.1. According to Figure 9, for the contrast *C*=2:1, the lightness difference is ΔL^*=1.7. The ranges of the lightness and the chromatic value is reduced to approximately 33%. The three-dimensional colour gamut is reduced to 0.333^3=0.037 or approximately 4% of the colour gamut for C=25:1. According to Figure 9, the visual properties apply:

Even with the contrast C=2:1, 16 grey and chromatic steps are distinguishable.

9. Change of chromatic value and hue angle threshold for contrasts C=25:1 and 2:1



DES91–7N

Figure 10: Spectral and chromatic values of six *Ostwald* colours for the CIE illuminant E with the hue discrimination as a function of hue angle Δh_{A2B2} for the contrast 2:1.
For the download of this figure, see http://color.li.tu-berlin.de/DES9/DES91-7N.PDF

Figure 10 shows in the middle the colour values, which are about a factor 3 smaller for the contrast C=2:1 than the chromatic values for the contrast C=25:1, see Figure 7. The corresponding reduction of the lightness discrimination by a factor 3 is shown in Figure 9.

For the hue discrimination ΔH_{A2B2} it is valid:

$$\Delta H_{A2B2} = C_{A2B2}\ \Delta h_{A2B2}.\ [9]$$

In Fig. 10, the chromatic value C_{A2B2} is smaller by a factor 3 compared to Fig. 8. Therefore, the hue angle difference Δh_{A2B2}=18 is greater by a factor 3. Accordingly, the wavelength difference Δλ increases by a factor 3 compared to the Fig. 7 for the six *Ostwald* colour spectra of Fig. 10.

The signals of complementary *Ostwald* colours, such as R^+, R^-, C^+ and C^-, are also smaller by a factor of 3 in Fig. 10 compared to Fig. 7.

Fig.10 also shows hue planes (green) with 5-step colour series (index i=0 to 4). For example, the index wi=4 indicates the whitish step w4. Accordingly, n4 is the blackish (n=noir) step n4.

Note: The calculation of the four elementary colours $RYGB_e$ from the six *Ostwald* device colours $RYGCBM_d$ (d=device), will be dealt in a later work . For example the elementary red R_e is defined according to the *visual criterion neither yellowish nor bluish*, see ISO/IEC 15775:2022.

10. Data transfer from *XYZ* to *LMS* with the *Hunt Pointer Esteves (HPE)* equations

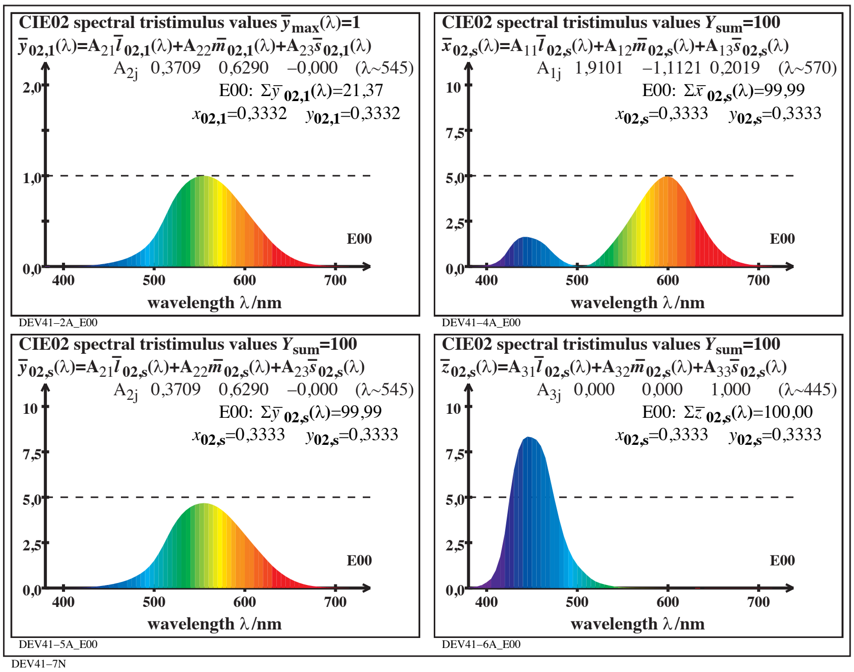


Bild 11: *HPE* transfer from *LMS* to *XYZ* for the CIE illuminant E and 2-degree observer

For the download of this figure, see http://color.li.tu-berlin.de/DEV4/DEV41-7N.PDF

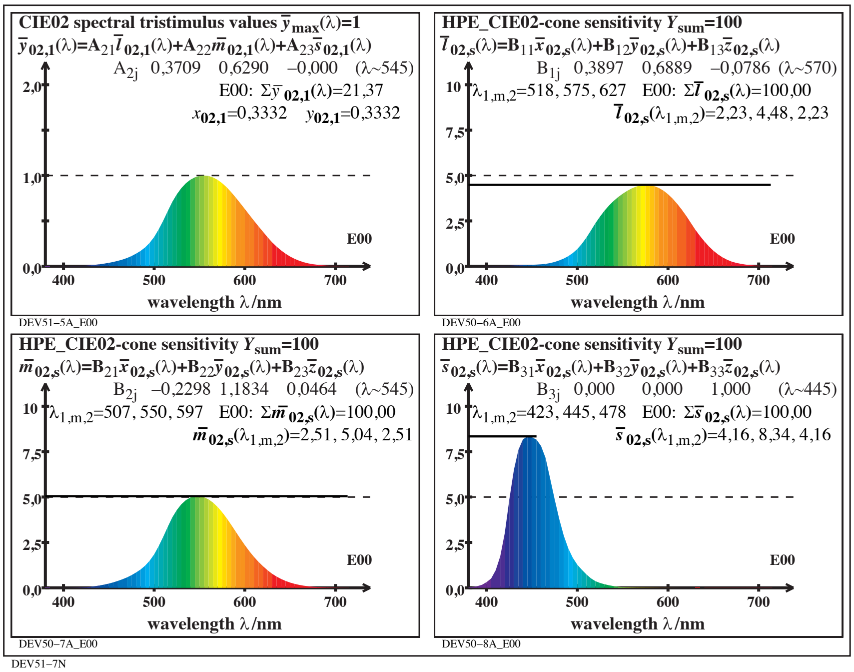


Figure 12: *HPE* transfer from *XYZ* to *LMS* for CIE illuminant E and 2-degree observer

For the download of this figure, see http://color.li.tu-berlin.de/DEV5/DEV51-7N.PDF

In Figure 11, all spectral CIE colour values are normalized to the sum of 100. The maximum is different. In the upper left image of Fig. 11, the spectral CIE colour value of 555 nm is normalized to one. This curve is called the spectral luminous sensitivity $V(\lambda)$.

Figure 12 contains the *Hunt-Pointer-Estevez (HPE)* equations, see CIE 015:2018, equation D.9. Figure 11 shows the *inverse* equations, see CIE 015:2018, equation D.59. The *HPE* equations in Figure 12 transform the spectral CIE tristimulus values to the spectral *LMS (PDT)* cone sensitivities. The spectral sum is again normalized to 100.

12. Data *CIEXYZ*, *TUBL*ABCh*$_{A2B2}$ and *CIELMS (PDT)* for *Ostwald*-optimal colours

Ostw data *rgb*, XYZxy*, and *L*ABCh*$_{AB2}$ in L*AB2JND-colour space											
Tristimulus values of black and white: Y_N=0,0, Y_W=88,6											
		CIEXYZ data					***L*ABCh*$_{AB2}$ data**				
	*rgb**	X_d	Y_d	Z_d	x_d	y_d	L^*_d	$A_{2,d}$	$B_{2,d}$	$C_{AB2,d}$	$h_{AB2,d}$
R$_d$	**1 0 0**	55,28	36,99	0,67	0,594	0,397	67,26	55,67	31,68	64,06	**29**
Y$_d$	**1 1 0**	67,93	72,65	1,12	0,479	0,512	88,28	18,96	62,39	65,21	**73**
G$_d$	**0 1 0**	21,11	57,87	13,29	0,228	0,627	80,66	−61,74	39,78	73,45	**147**
C$_d$	**0 1 1**	28,91	51,60	95,79	0,163	0,292	77,04	−55,69	−31,68	64,07	**209**
B$_d$	**0 0 1**	16,26	15,93	95,34	0,127	0,124	46,89	−18,96	−62,38	65,20	**253**
M$_d$	**1 0 1**	63,08	30,71	83,17	0,356	0,173	62,26	61,72	−39,78	73,43	**327**
N$_d$	**0 0 0**	0,00	0,00	0,00	0,333	0,333	0,08	0,00	0,00	0,00	**0**
W$_d$	**1 1 1**	84,21	88,60	96,48	0,312	0,329	95,41	0,00	0,00	0,00	**0**
N1$_d$	**0,00**	0,00	0,00	0,00	0,333	0,333	0,08	0,00	0,00	0,00	**0**
W1$_d$	**1,13**	95,05	100,01	108,30	0,313	0,329	100,00	0,16	0,48	0,51	**71**
Z1$_d$	**0,18**	17,10	17,99	19,49	0,313	0,329	49,48	0,03	0,08	0,09	**64**

DEV20−7N

Fig. 13. CIE and TUB data of the *Ostwald* colours *(Ostw)* for the CIE illuminant D65, the CIE 2-degree observer and the normalization Y_W=88.60 according to ISO 9241-306.
For the download of this figure, see http://color.li.tu-berlin.de/DEV2/DEV20-7N.PDF

Ostw data *rgb*, XYZxy*, and *L* & LMS(PDT)*										
Tristimulus values of black and white: Y_N=0,0, Y_W=88,6										
		CIEXYZ data D65−>E					***L* & LMS(PDT)* data**			
	*rgb**	X_d	Y_d	Z_d	x_d	y_d	L^*_d	L_P	M_D	S_T
R$_d$	**1 0 0**	58,16	36,99	0,61	0,607	0,386	67,26	48,10	30,43	0,61
Y$_d$	**1 1 0**	71,47	72,65	1,03	0,492	0,500	88,28	77,82	69,59	1,03
G$_d$	**0 1 0**	22,21	57,87	12,20	0,240	0,627	80,66	47,56	63,94	12,20
C$_d$	**0 1 1**	30,41	51,60	87,96	0,178	0,303	77,04	40,48	58,15	87,96
B$_d$	**0 0 1**	17,11	15,93	87,55	0,141	0,132	46,89	10,76	18,99	87,55
M$_d$	**1 0 1**	66,36	30,71	76,37	0,382	0,177	62,26	41,02	24,64	76,37
N$_d$	**0 0 0**	0,00	0,00	0,00	0,354	0,336	0,08	0,00	0,00	0,00
W$_d$	**1 1 1**	88,60	88,60	88,60	0,333	0,333	95,41	88,60	88,60	88,60
N1$_d$	**0,00**	0,00	0,00	0,00	0,354	0,336	0,08	0,00	0,00	0,00
W1$_d$	**1,13**	100,00	100,01	99,45	0,333	0,333	100,00	100,05	99,98	99,45
Z1$_d$	**0,18**	17,99	17,99	17,89	0,333	0,333	49,48	18,00	17,98	17,89

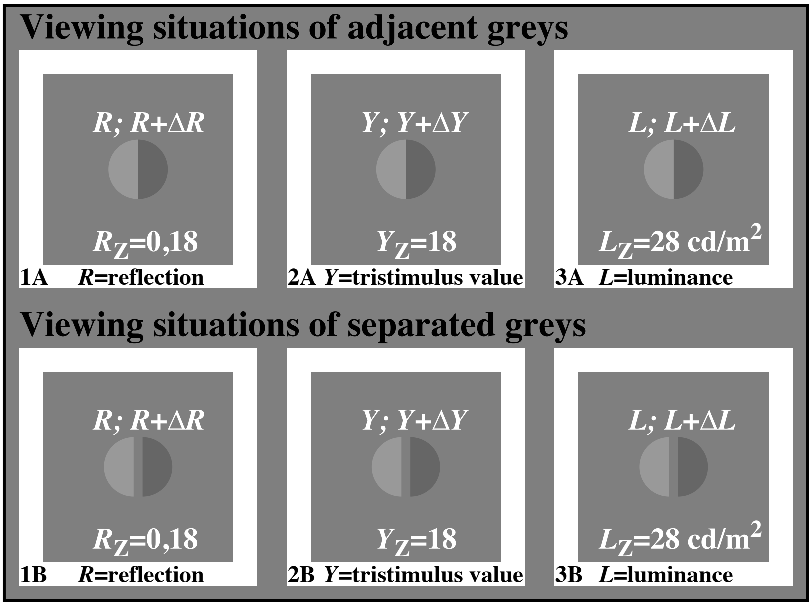
DEV30−7N

Figure 14. CIE and LMS data of the *Ostwald colours* (*Ostw*) for the CIE illuminant (D65 ->E compared to Fig. 13) with *HPE-LMS* data calculated according to CIE 015:2018
For the download of this figure, see http://color.li.tu-berlin.de/DEV3/DEV30-7N.PDF

The Figures 13 and 14 show the relationships between the different colour coordinates, for example between *rgb**, *CIEXYZ, TUBLAB,* and *LMS(PDT).* The indices P=Protanop, D=Deuteranop and T=Tritanop describe the three colour vision deficiencies.

The LMS values of the antagonistic colour pairs *R-C, Y-B* and *G-M* sum to the *LMS* values of white W_d. These and combinations for applications are still being discussed.

13 Viewing situations with data: reflection *R*, CIE-tristimulus value *Y*, and luminance *L*



CEA10–2N

Figure 15. Adjacent (top) and separate (bottom) grey samples, defined by the standard values for the reflection *R*, the CIE-tristimulus value *Y*, and the luminance *L*.
For the download of this figure, see http://color.li.tu-berlin.de/CEA1/CEA10-2N.PDF

According to Figure 15, one can use the reflection *R*, the CIE-tristimulus value *Y*, or the luminance *L* for the calculation of the contrast *C*. All are proportional and therefore the ratio for the contrast values of *R*/*ΔR* or *Y*/*ΔY* or *L*/*ΔL* is the same. In this work, the reflection *R* would usually be used. This avoids a difficulty with *CIEXYZ* data. These data depend on the illuminant and the adaptation. For many users they are difficult to understand, for example in the field of design. The described applications of the ISO-test charts allow a playful introduction to the field of colour without *XYZ* values.

At the visual threshold, the measured differences *ΔR*, *ΔY* or *ΔL of separated* samples increase by at least a factor of two compared to the differences for adjacent samples, see *Kittelmann, Philipp (2010*), *Visuelle Beurteilung von kleinen und großen Farbdifferenzen und Beschreibung mit Farbabstandsformeln, Dissertation*, *Technische Universität Berlin, Fakultät IV, Elektrotechnik und Informatik (132 pages, 9.8 MB, PDF format), see* http://opus.kobv.de/tuberlin/volltexte/2010/2634/.

For many applications the colour perception with variable geometric and temporal presentation and viewing with many contrast ratios needs more research. This work and video conferences try to describe basic characteristics and applications, see for example the colour loop in Fig. 1.

Note: For topics and dates of video meetings about the TUB-relativity model of colour vision for light and surface colours, and applications, see the TUB website:
http://farbe.li.tu-berlin.de/index.html
This website includes information for the registration and for individual email information.

Acknowledgements: I would like to thank *Prof. Florian Süßl* (BHT Berlin), *Detlef Ruschin* (HHI Berlin) and *Thorstein Seim* (University of Oslo) for their contributions to improving this work. Further parts of the *TUB-relativity model of colour vision for light and surface colours* are in preparation, see under publications: http://color.li.tu-berlin.de/XY91FEN.html